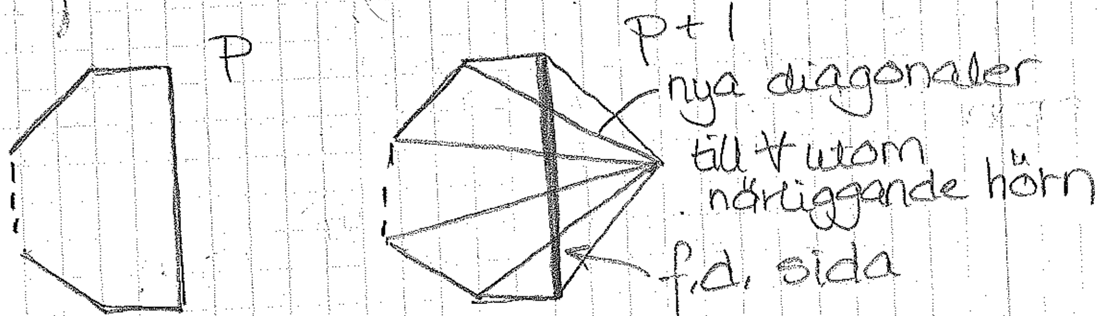
1246. $d_n = \frac{n(n-3)}{2}$

① $n=3 \Rightarrow \frac{3(3-3)}{2} = 0$ V.S.V.

② Antag att: $d_p = \frac{p(p-3)}{2}$

③ $n = p+1$ $d_{p+1} = \frac{(p+1)(p+1-3)}{2} = \frac{(p+1)(p-2)}{2}$

Lägger vi till ett hörn i en p-hörning får vi p-2 nya diagonaler och en sida i p-hörningen blir diagonal i (p+1)-hörningen.



$\therefore$ $d_{p+1} = d_p + (p-2) + 1 = d_p + p - 1$

Detta ger:

$$\frac{(p+1)(p-2)}{2} = \frac{p(p-3)}{2} + (p-1)$$

$$\frac{p^2-2p+p-2}{2} = \frac{p^2-3p+2p-2}{2}$$

$$\frac{p^2-p-2}{2} = \frac{p^2-p-2}{2}$$ V.S.V.

1247. $\cos x + \cos 3x + \cos 5x + \ldots + \cos(2n-1)x = \dfrac{\sin 2nx}{2\sin x}$

där $\sin x$ är definierad dvs $x \neq \pi \cdot m$ m heltal

(1) $n = 1 \Rightarrow$ $\cos x = \dfrac{\sin 2x}{2\sin x}$

$2\sin x \cos x = \sin 2x$ v.s.v. (känt)

(2) Antag att

$$\cos x + \cos 3x + \ldots + \cos(2p-1)x = \frac{\sin 2px}{2\sin x}$$

(3) $n = p+1 \Rightarrow$

$$\cos x + \cos 3x + \ldots + \cos(2(p+1)-1)x = \frac{\sin 2(p+1)x}{2\sin x}$$

$$\cos x + \cos 3x + \ldots + \cos(2p+1)x = \frac{\sin 2(px+x)}{2\sin x}$$

$$\cos x + \cos 3x + \ldots + \cos(2p-1)x + \cos(2p+1)x = \frac{\sin 2(p+1)x}{2\sin x}$$

$\left\{\begin{matrix}\text{enl.}\\ \text{ind.}\\ \text{ant.}\end{matrix}\right\}$ $\dfrac{\sin 2px}{2\sin x} + \cos(2p+1)x = \dfrac{\sin 2(p+1)x}{2\sin x}$

$$\frac{\sin 2px + 2\sin x\cos(2p+1)x}{2\sin x} = \frac{\sin 2(p+1)x}{2\sin x}$$

$\left\{\sin x \cos(2p+1)x = \frac{1}{2}\left(\sin(x+2px+x) + \sin(x-2px-x)\right)\right\}$

$$\frac{\sin 2px + \sin 2x(p+1) + \sin(-2px)}{2\sin x} = \frac{\sin 2x(p+1)}{2\sin x}$$

$\left\{\sin(-2px) = -\sin 2px\right\}$

$$\frac{\cancel{\sin 2px} + \sin 2x(p+1) - \cancel{\sin 2px}}{2\sin x} = \frac{\sin 2x(p+1)}{2\sin x}$$

$\therefore$ $\dfrac{\sin 2x(p+1)}{2\sin x} = \dfrac{\sin 2x(p+1)}{2\sin x}$ v.s.v.