26. a) $y' = -0,121 \cdot 10^{-3} y$

$y' + 1,21 \cdot 10^{-4} y = 0$

$y = C \cdot e^{-1,21 \cdot 10^{-4} t}$ ①

b) $y(0) = 5,41 \cdot 10^{16}$ ins. i ① ⇒

$C \cdot e^0 = 5,41 \cdot 10^{16}$

$C = 5,41 \cdot 10^{16}$

$\therefore y = 5,41 \cdot 10^{16} \cdot e^{-1,21 \cdot 10^{-4} t}$

$y(2000) \approx \underline{4,2 \cdot 10^{16} \text{ atomer}}$

c) Halveringstiden $t_{1/2}$?

$5,41 \cdot 10^{16} \cdot e^{-1,21 \cdot 10^{-4} t_{1/2}} = 0,5 \cdot 5,41 \cdot 10^{16}$

$\underline{t_{1/2} \approx 5730 \text{ år}}$

25. c) $y(10) = 20 - 19e^{-0,1 \cdot 10} = 20 - 19e^{-1}$ ①

$y'(10) = -0,1(y - 20)$ (givet) ins ① ⇒

$y'(10) = -0,1(20 - 19e^{-1} - 20) =$

$= \frac{1,9}{e} \approx 0,70 \text{ grader/min}$

24. $N(t)$ = antal myror vid tiden t

$N'(t) = k \cdot N(t)$

$N' - kN = 0$

$N = Ce^{kt}$ $\quad N(0) = N_0 \Rightarrow C = N_0$

$N = N_0 e^{kt}$ $\quad N(t_2) = 2N_0$

$N_0 e^{kt_2} = 2N_0$

$t_2 = \frac{\ln 2}{k}$

23. $\frac{dy}{dx} = -\frac{y}{200}$ $\quad y$: mg/l nitrat $\quad x$ = l H_2O

Vi sätter $x=0$ vid $y = 100$ mg/l

$\frac{dy}{dx} = -\frac{1}{200}y$

$\frac{dy}{dx} + \frac{1}{200}y = 0$

$y = Ce^{-x/200}$ $\quad y(0) = 100 \Rightarrow C = 100$

$y = 100e^{-x/200}$ $\quad x = ?$ då $y = 75$ mg/l

$75 = 100e^{-x/200}$

$x \approx 57$ l

22. $K' + 0{,}1K = 450$ (1) $K(0) = 600$

a) $K = ?$ $\quad K = K_p + K_h$

Partikulärlösen.

Ansätt $K_p = a$ $\quad K_p' = 0$ ins. i (1)

$0 + 0{,}1a = 450$

$a = 4500$

Homogen lösen.

$K_h' + 0{,}1K_h = 0$

$K_h = Ce^{-0{,}1t}$

Begynnelsevillkor

$K(0) = 600$

$K = K_p + K_h = 4500 + Ce^{-0{,}1t}$ (2) $\Big\} \Rightarrow$

$600 = 4500 + Ce^0$

$C = -3900$ ins. i (2) $\Rightarrow$

$K = 4500 - 3900e^{-0{,}1t}$

b) $K(4) = 4500 - 3900e^{-0{,}1 \cdot 4} \approx \underline{1886 \text{ k€}}$ ← eller €

21. $y = e^{-kx^2}$ $\quad k = ?$ om $y'' + 2kxy' + y = 0$ (1)

$y' = -k \cdot 2x \cdot e^{-kx^2}$ (anv. $\frac{d}{dx} f(x) \cdot g(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$)

$y'' = -k \cdot 2 \cdot e^{-kx^2} - k2x \cdot (-k2x e^{-kx^2}) =$

$= -2ke^{-kx^2} + 4k^2x^2 e^{-kx^2}$

Sätt in i (1):

$-2ke^{-kx^2} + 4k^2x^2 e^{-kx^2} + 2kx(-k2x e^{-kx^2}) + e^{-kx^2} = 0$

$-2ke^{-kx^2} + 4k^2x^2 e^{-kx^2} - 4k^2x^2 e^{-kx^2} + e^{-kx^2} = 0$

$-2ke^{-kx^2} + e^{-kx^2} = 0$

$-2k + 1 = 0$

$k = \frac{1}{2}$

20. $y(3) = ?$ $\quad y' = 3y - x$ $\quad h = 0,5$

$y(1) = 1$

$y'(1) = 3 \cdot 1 - 1 = 2$

$y(1,5) = 2 \cdot 0,5 + 1 = 2$

$y'(1,5) = 3 \cdot 2 - 1,5 = 4,5$

$y(2) = 4,5 \cdot 0,5 + 2 = 4,25$

$y'(2) = 3 \cdot 4,25 - 2 = 10,75$

$y(2,5) = 10,75 \cdot 0,5 + 4,25 = 9,625$

$y'(2,5) = 3 \cdot 9,625 - 2,5 = 26,375$

$y(3) = 26,375 \cdot 0,5 + 9,625 = \underline{22,8}$

19. $y' - 2y = 4x^2$ (1)

$y = y_p + y_h$ (2)

Partikulärlösen:

Ansätt $y_p = ax^2 + bx + c$ $\quad y_p' = 2ax + b$

ins i (1):

$2ax + b - 2(ax^2 + bx + c) = 4x^2$

$2ax + b - 2ax^2 - 2bx - 2c = 4x^2$

(x^2): $-2a = 4$

$a = -2$

(x): $2a - 2b = 0$ $\quad \{a = -2\}$

$-4 = 2b$

$b = -2$

(x^0): $b - 2c = 0$ $\quad \{b = -2\}$

$-2 = 2c$

$c = -1$

$\therefore y_p = -2x^2 - 2x - 1$ (3)

Homogen lösen:

$y_h' - 2y_h = 0$

$y_h = Ce^{2x}$ (4)

(2), (3) & (4) ger:

$y = -2x^2 - 2x - 1 + Ce^{2x}$